Take It to the Limit: The Power of the Infinite

Key Idea of Calculus

- Make a series of better & better _____ that converge to the desired quantity as a ____.

We have seen this before:

0.9

0.99

0.999

0.9999

⋮

Limit? One one hand: ____

On the other hand: ____

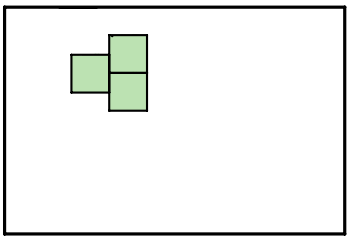
This is what we mean when we say _____.

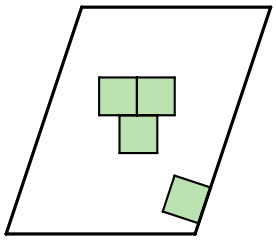
It is also important for understanding the number ____, which is important for _____ compounded interest.

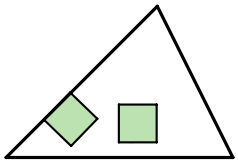
To illustrate the concept of a limit, we look at Archimedes' strategy for finding the area of a ____.

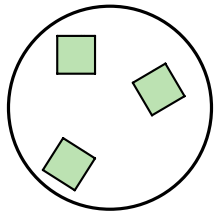
Areas of shapes

How many 1×1 squares does it take to cover the shape exactly, with no overlap and no squares sticking out?

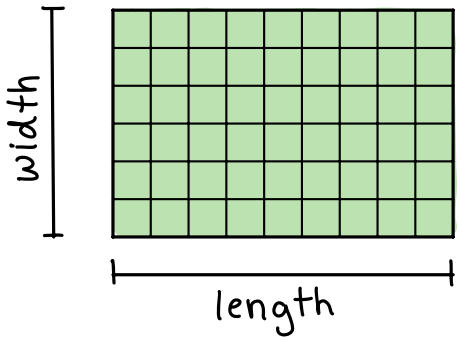








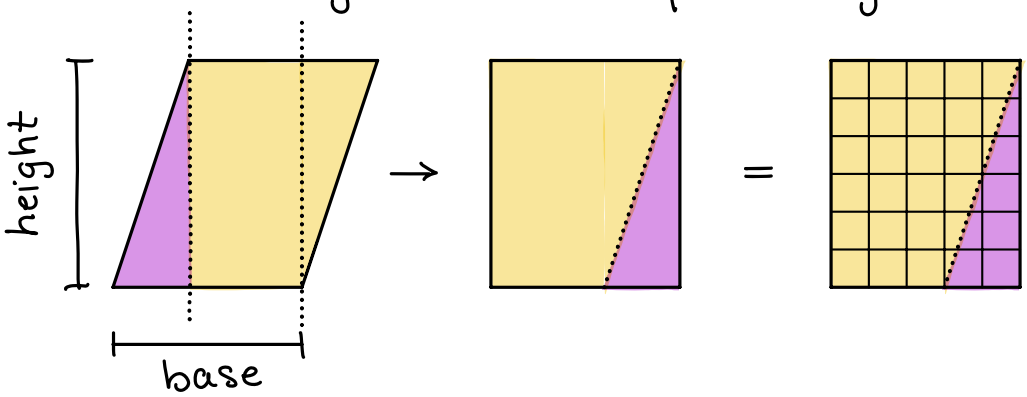
Rectangles ✓



area = (# squares covering shape)

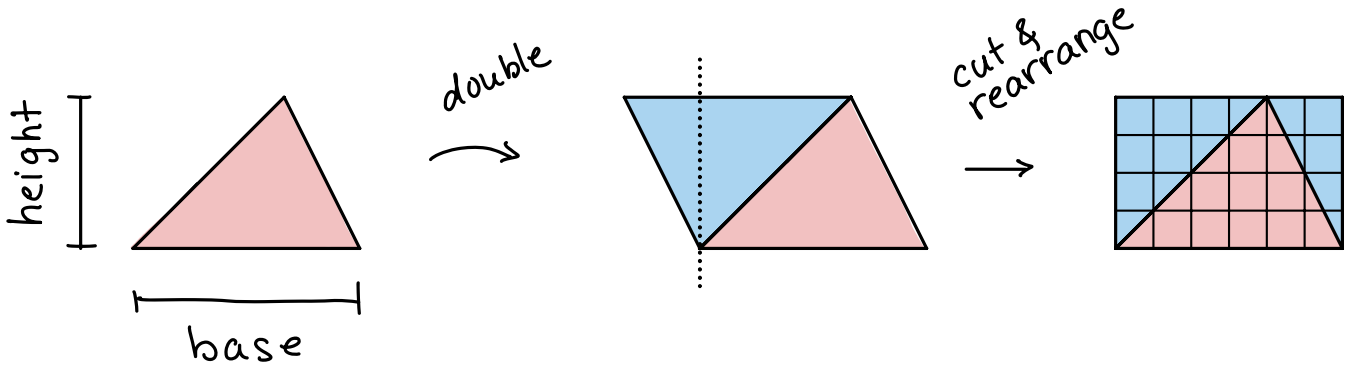
(area of rectangle) = (____) × (____)

Cut & rearrange other shapes to get rectangles ...

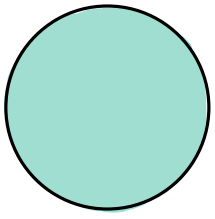


rectangle!

(area of parallelogram) = (____) × (____)

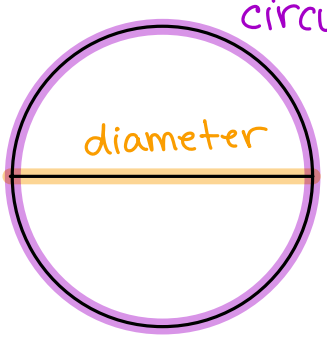


(area of triangle) = __ (base) × (height)



Cut up & rearrange?
Pieces will always have curved parts...

Circles & the Number π



$$\pi = \frac{\text{circumference}}{\text{diameter}}$$

Ratio is the ____ for all circles.

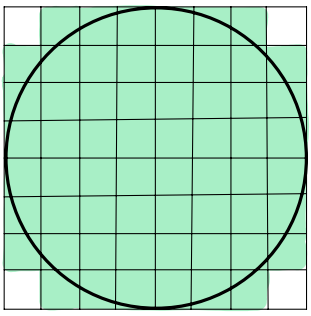
Like $\sqrt{2}$, the number π is _____ (Lambert, 1760s).

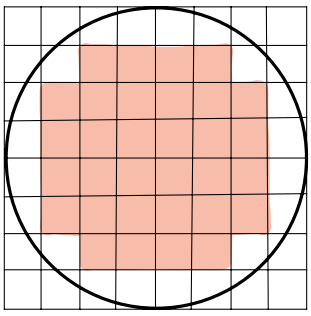
Today, _____ digits of π are known (Solidigm, 2024)

But _____ is good enough for odometer & speedometer.

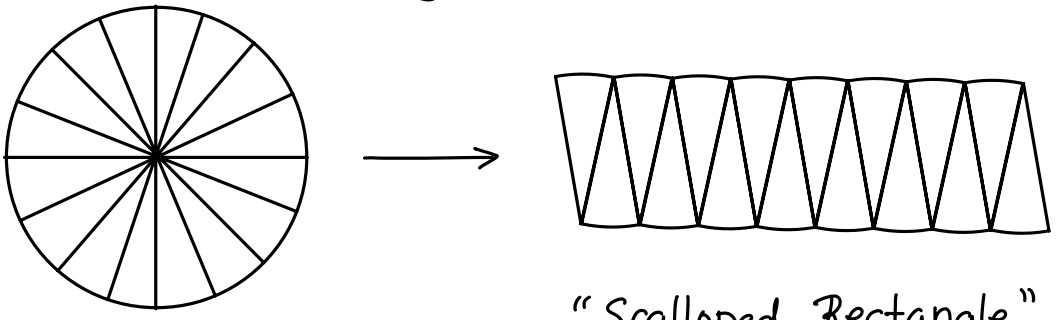
NASA uses ___ digits past the decimal point.

Back to the area of a circle.





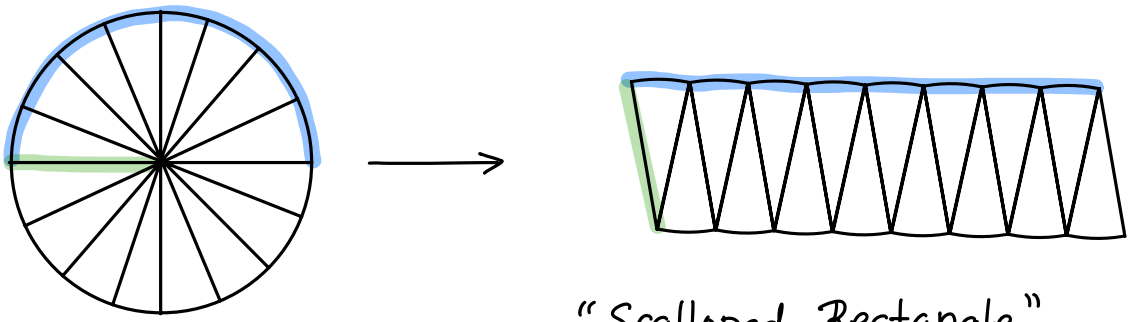
Archimedes' Strategy: Cut into wedges & rearrange.



"Scalloped Rectangle"

More & more wedges... scalloped rectangle becomes more & more like a rectangle, whose area we can compute!

After the Activity:



"Scalloped Rectangle"

Area of Circle of radius r = ☐ × ☐ = ___

Archimedes' Estimate For π:

perimeter of _____ polygon $< 2\pi r <$ perimeter of _____ polygon

96-sided polygon: $3\frac{10}{71} < \pi < 3\frac{1}{7}$

$3.1408 < \pi < 3.1429$

Approximation & iteration... modern field of numerical analysis
- engineer cars that are optimally streamlined
- simulations for chemotherapy drugs & cancer cells

⋮

economics ... technology ...